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Parametric studies on pulsating heat pipe

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Abstract

Purpose – The purpose of this paper is to present a numerical investigation on pulsating heat pipe (PHP) to study the slug velocities as a function of various parameters.

Design/methodology/approach – The governing equation of PHP is solved using explicit embedded Runge-Kutta method, the Dormand–Prince pair in conjunction with MATLAB with the nomenclature 45 for the determination of displacement and the velocity of the slug.

Findings – The results show that lower fill ratio, higher diameter, higher operating temperature and higher temperature difference between evaporator and condenser for a given working fluid results in higher slug velocities, indicating higher momentum transfer and hence better heat transport.

Research limitations/implications – Under steady state conditions, the design of a PHP is facilitated through the introduction of non-dimensional numbers.

Originality/value – The displacement and slug velocities for additional working fluids, namely ethanol and methanol, are determined for the first time. The behaviour of non-dimensional numbers, i.e. Poiseuille number, capillary number and Eckert number in a PHP as a function of various parameters have been studied for the first time.

Keywords Heat transfer, Pipes, Pulsating flow

Paper type Research paper

Nomenclature

A	tube cross sectional area (m^2)
Ca	capillary number (dimensionless)
C_p	specific heat at constant pressure (J/kgK)
D	diameter (m)
Ec	Eckert number (dimensionless)
F	force (N)
Fo	Fourier number, dimensionless
g	gravitational acceleration (m/s^2)
h_{fg}	latent heat of vaporization (kJ/kg)
L	length (m)
PHP	pulsating heat pipe
ΔP	pressure difference (N/m^2)
Po	Poiseuille number, dimensionless
R	gas constant (J/kgK)
T	temperature (K)
Δ	temperature difference between the evaporator and condenser (K)
t	time (s)
V	dx/dt = velocity of the slug (m/s)
x	displacement (m)

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Greek symbols

- μ viscosity (N-s/m^2)
- γ kinematic viscosity (m^2/s)
- ρ density (kg/m^3)
- ϕ filled liquid ratio
- ω frequency (rad/s)
- σ surface tension (N/m)

Subscripts

- a adiabatic
- c condenser
- d driving
- e evaporator
- i inertia
- l liquid
- max maximum
- min minimum
- vp vapour pressure
- v vapour
- vi viscous

1. Introduction

Thermal management is the challenge of the day in electronic product development. One of the devices used to remove the heat effectively from the heat source is the heat pipes (Faghri, 1995). The oscillating or pulsating heat pipe (PHP) first proposed by Akachi (1990) is another promising heat transfer device for applications like electronic cabinet cooling. It is simple in structure with a small diameter coil filled with certain working fluid in it and extended from the heat source to sink. PHP does not contain any wick structure to return the condensate back to the heating section unlike a common heat pipe. Instead, PHP uses the technique of transporting the working fluid by means of differential pressure across vapour plugs from evaporator to condenser and back. The vapour formed at the evaporator is pushed towards the condenser in the form of discrete vapour bubbles. The vapour gets partially condensed at the condenser and loses the heat and returns to evaporator to complete the cycle. Since PHP is a passive device which makes use of the heat of heat source to drive the vapour plugs and operate in cyclic mode, it is gaining attention of many investigators for possible application of electronic cabinet cooling.

Both experimental and numerical investigations on PHP's and their performance are reported in the literature though not in sufficient numbers. The experimental investigations mainly focus on flow visualization and measurement of temperature (Charoensawan *et al.*, 2003; Khandekar *et al.*, 2003). Numerical investigations deal with various PHP models and performance evaluation through computations. (Shafii *et al.*, 2001; Shafii and Faghri, 2002; Zhang *et al.*, 2002; Zhang and Faghri, 2003; Ma *et al.*, 2006, 2008; Khrustalev and Faghri, 1994). However, a comprehensive model dealing with the effects of several parameters on the design of PHP is still lacking.

Thermal modelling of vertically placed unlooped and looped PHP with three heating sections and two cooling sections was presented by Shafii *et al.* (2001). The dimensional governing equations of mass, momentum and energy were solved using an explicit scheme. They observed that the number of vapour plugs is reduced to the number of heating sections no matter how many vapour plugs were initially in the PHP. Zhang and Faghri (2002) studied the heat transfer phenomena in the evaporator and condenser sections of a PHP with open end for analysing thin film evaporation and condensation. The heat transfer solutions were applied to the thermal model of the PHP and a parametric study was presented. Zhang *et al.* (2002) developed the empirical correlations of amplitude and circular frequency of oscillations for liquid vapour pulsating flow in a vertically placed U-shaped miniature tube using momentum and energy equations. The authors showed that the initial displacement of the liquid slug has a marginal effect on the amplitude and angular frequency of oscillation. In the above works, heat transfer in PHP is studied considering the pressure difference between evaporator and condenser as the driving force. A mathematical model which deals with the oscillating motion of the fluid in a PHP was proposed by Ma *et al.* (2006) based on the temperature difference between the evaporator and condenser as the driving force. This model established the relation between oscillating frequency and geometry, thermal potential, fill ratio, working fluid and operating temperature. The results of their study are used to understand the mechanism governing the pulsating phenomenon in a PHP. The authors relate the pressure difference between evaporator and condenser with the temperature difference using Clausius-Clapeyron equation. The model was solved for the displacement of the slug and highlights the characteristics of PHP in the saturation region. The authors considered water and Acetone as the working fluids in their study.

The present study extends the Ma *et al.*'s (2006) work further by determining the displacement for additional working fluids. The slug velocities are also determined from the solution of the governing differential equation. The effect of driving and opposing forces are analysed. The pulsating flow of the fluid in the heat pipe is studied with the help of three non-dimensional numbers, namely, Poiseuille number, capillary number and Eckert number which can be considered as the important parameters in the design of PHP.

2. Governing differential equation

The model proposed by Ma *et al.* (2006) considers a PHP with evaporator, condenser and adiabatic section. On addition of heat to the evaporating section, the saturated liquid is converted to saturated vapour. Figure 1 shows the physical model used for obtaining the governing differential equation. The characteristic length of PHP consists of the length of evaporator section L_e, adiabatic section L_a and condenser section L_c. Hence,

$$L = L_e + L_a + L_c. \tag{1}$$

As the fluid flows in a PHP, the fluid gets evaporated in the evaporator and condensed in the condenser which results in the volume expansion and contraction of the bubbles. This causes an oscillating motion which affects the saturation temperature in the evaporator and condenser section. If the maximum and minimum temperature difference between the evaporator and condenser section are ΔT_{max} and ΔT_{min}, respectively, then the temperature difference between the evaporator and condenser section will vary between ΔT_{max} and ΔT_{min} and is given by $\Delta T_{max} - \Delta T_{min}/2$. Considering the oscillating nature of PHP and system oscillation frequency as ω, the thermal driving potential can be written as:

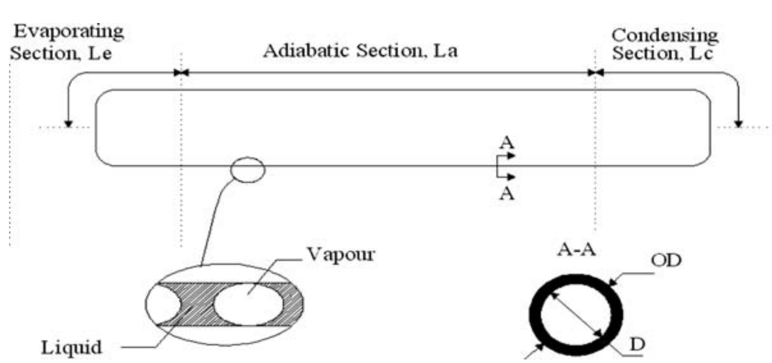


Figure 1.
PHP considered in the present study

$$\Delta T = \frac{\Delta T_{\max} - \Delta T_{\min}}{2}[1 + \cos(\omega t)]. \quad (2)$$

Using Clausius-Clapeyron equation, the pressure difference between evaporator and condenser can be related to the thermal driving potential as:

$$\Delta p = \Delta T \frac{h_{fg}\rho_e}{T_e}. \quad (3)$$

Thus, the driving force causing the pulsating motion in a PHP is expressed as:

$$F_d = \left(\frac{A h_{fg}\rho_e}{T_e}\right)\left(\frac{\Delta T_{\max} - \Delta T_{\min}}{2}\right)[1 + \cos(\omega t)]. \quad (4)$$

This driving force overcomes:

- the viscous force which arises due to the interaction between liquid/vapour and the pipe walls;
- the force due to vapour pressure which arise due to volume contraction and expansion of bubbles; and
- the force due to inertia.

From Newton's law, the governing equation for fluid flow (Ma *et al.*, 2006) in a PHP is:

$$(\rho_l L_l + \rho_v L_v)A\frac{d^2x}{dt^2} + \left[32\left(\frac{\mu_l L_l + \mu_v L_v}{D^2}\right)\right]A\frac{dx}{dt} + \frac{A\rho_v RT}{L_v}x = \left(\frac{A h_{fg}\rho_v}{T_e}\right) \times \left(\frac{\Delta T_{\max} - \Delta T_{\min}}{2}\right)[1 + \cos(\omega t)]. \quad (5)$$

The inertia force, the viscous force and the force due to vapour pressure in the above equation are given by:

$$F_i = (\rho_l L_l + \rho_v L_v)A\frac{d^2x}{dt^2}, \quad (6)$$

$$F_{vi} = \left[32\left(\frac{\mu_l L_l + \mu_v L_v}{D^2}\right)\right]A\frac{dx}{dt}, \quad (7)$$

$$F_{vp} = \frac{A\rho_v RT}{L_v}x. \quad (8)$$

The governing Equation (5) for fluid flow in a PHP is similar to the governing equation of forced damped mechanical vibration with the following initial conditions:

$$x = 0 \quad \text{and} \quad \frac{dx}{dt} = 0 \text{ at } t = 0. \quad (9)$$

Ma *et al.* (2006) have used Laplace transformation to solve Equation (5) whereas in the present study, the embedded Runge–Kutta formula as given in MATLAB with the nomenclature ODE 45 is used to solve for both slug displacement and slug velocity.

3. Highlights of embedded Runge-Kutta method used

In the present study, the explicit embedded Runge-Kutta method is used to solve the differential Equation (5) for slug displacement and slug velocity. In general, the Runge-Kutta methods possess the advantage of requiring only the function values at some selected points on the sub interval. The code as given in MATLAB with the nomenclature ODE 45 (Ashino *et al.*, 2000) which is based on the explicit embedded Runge-Kutta formula, the Dormand and Prince pair (1980) is used to solve Equation (5) to obtain the slug displacement and slug velocity.

Embedded pairs of Runge-Kutta methods of orders p and p + 1 with interpolant are used to determine the local error $E_{n+1} = x_{n+1} - \hat{x}_{n+1}$ where x_{n+1} corresponds to the higher order solution (i.e. at p + 1) and $\hat{x}_{n+1}$ corresponds to lower order solution (i.e. at p). This is used to monitor the local error and hence control the step size. For example, the formulae $h_{n+1} = 0.9h_n\left[\delta/\|E_{n+1}\|_\infty\right]^{1/p+1}$ (error per step control) and $h_{n+1} = 0.9h_n\left[\delta/\|E_{n+1}/h_n\|_\infty\right]^{1/p}$ (error per unit step control) are widely used, δ being the maximum allowable local error and h_n being the time step (Dormand and Prince, 1980). Thus, the time discretization in this method is based on the local error estimate mentioned above.

The embedded Runge-Kutta algorithm can be written as:

$$x_{n+1} = \hat{x}_n + h_n\varphi(\hat{x}_n, h_n) = \hat{x}_n + \sum_{i=1}^{s} b_i k_i, \qquad (10)$$

$$\hat{x}_{n+1} = \hat{x}_n + h_n\hat{\varphi}(\hat{x}_n, h_n) = \hat{x}_n + \sum_{i=1}^{s} \hat{b}_i k_i, \qquad (11)$$

where:

$$k_1 = h_n f(\hat{x}_n), k_i = h_n f(\hat{x}_n) + \sum\nolimits_{j=1}^{i-1} a_{ij}k_j \quad i = 2, 3, \ldots, s,$$

in which s = number of stages.

The popular pair of embedded Runge-Kutta method of order five and four with interpolant due to Dormand and Prince is given in the form of a Butcher table (2003).

In Table I, the components of the vector c_i are the increments of t_n and the entries of the matrix a_{ij} are the multipliers of the approximate slopes which after multiplication by the step size h_n, increment x_n. The components of the vector b are the weights in the combination of the intermediary values k_j. Number 5 in the designation DP5(4)7M means that the solution is advanced with the solution x_{n+1} of order five. Number 4 in the parentheses means that the solution $\hat{x}_{n+1}$ of order four is used to obtain the local error estimate. Number 7 means that the method has seven stages. The letter M means that the constant c_6 in the top order error term has been minimized while maintaining stability. Six stages are necessary for the method of order five. The seventh stage is necessary to have an interpolant. However, this is really a six stage method since the first step at t_{n+1} is the same as the last step at t_n i.e. $k_1^{(n+1)} = k_7^{(n)}$. Such methods are called First Step As Last.

In the present work, the governing Equation (5) is solved using the MATLAB ODE 45 code which is based on the Dormand and Prince pair DP5(4)7M principle explained above in this section. The governing Equation (5) is a second-order ordinary differential equation (ODE) in terms of independent variable, time t and dependent variable,

	c_i	a_{ij}						
k_1	0	0						
k_2	$\frac{1}{5}$	$\frac{1}{5}$	0					
k_3	$\frac{3}{10}$	$\frac{3}{40}$	$\frac{9}{40}$	0				
k_4	$\frac{4}{5}$	$\frac{44}{45}$	$-\frac{56}{15}$	$\frac{32}{9}$	0			
k_5	$\frac{8}{9}$	$\frac{19,372}{6,561}$	$-\frac{25,360}{2,187}$	$\frac{64,448}{6,561}$	$-\frac{212}{729}$	0		
k_6	1	$\frac{9,017}{3,168}$	$-\frac{355}{33}$	$\frac{46,732}{5,247}$	$\frac{49}{176}$	$-\frac{5,103}{18,656}$	0	
k_7	1	$\frac{35}{384}$	0	$\frac{500}{1,113}$	$\frac{125}{192}$	$-\frac{2,187}{6,784}$	$\frac{11}{84}$	
$\hat{x}_{n+1}$	$\hat{b}_i$	$\frac{35}{384}$	0	$\frac{500}{1,113}$	$\frac{125}{192}$	$-\frac{2,187}{6,784}$	$\frac{11}{84}$	0
x_{n+1}	b_i	$\frac{5,179}{57,600}$	0	$\frac{7,571}{16,695}$	$\frac{393}{640}$	$-\frac{92,097}{3,39,200}$	$\frac{187}{2,100}$	$\frac{1}{40}$
Error		$-\frac{71}{57,600}$	0	$\frac{71}{16,695}$	$-\frac{71}{1,920}$	$\frac{17,253}{3,39,200}$	$-\frac{22}{525}$	$\frac{1}{40}$

Source: Butcher (2003)

Table I. Butcher table of Dormand and Prince pair DP5(4)7M with interpolant

displacement x. In order to solve this second order differential Equation (5), it is written as a system of two first-order ODEs as:

$$x' = \frac{dx}{dt} = x_1 = V_s, \tag{12}$$

$$x_1' = \frac{d}{dt}(x_1) = \frac{d^2x}{dt^2} = \frac{\left(\frac{Ah_{fg}\rho_v}{T_e}\right)\left(\frac{\Delta T_{\max} - \Delta T_{\min}}{2}\right)[1 + \cos(\omega t)] - \left[32\left(\frac{\mu_l L_l + \mu_v L_v}{D^2}\right)Ax_1\right] - \left(\frac{A\rho_v RT}{L_v}x\right)}{(\rho_l L_l + \rho_v L_v)A}. \tag{13}$$

This splitting of Equation (5) into two first-order ODEs is required as MATLAB ODE 45 code based on the embedded Runge–Kutta formula can handle only the first-order ODEs.

The solution of Equation (5) using MATLAB ODE 45 is based on the following steps:

(1) Equation (5) is rewritten as a system of first-order ODEs as shown in Equations (12) and (13).

(2) Now, it is required to code it as a function that an ODE solver can use.

(3) Then the ODE solver, ODE 45 is called to solve Equations (12) and (13) for slug displacement and slug velocity.

For fill ratio = 50 per cent, diameter of PHP = 1.65 mm, operating temperature = 60 °C, total length of PHP = 304.8 mm, $\Delta T = 5$ K and with water being considered as the working fluid, Equation (13) reduces to the form of:

$$x_1' = \frac{d^2x}{dt^2} = 3.288 \times 10^{-5}[1 + \cos(\omega t)] - 1.21\text{x}10^{-5}x_1 - 1.8667 \times 10^{-3}x. \quad (14)$$

Equations (12) and (14) are solved according to the above-mentioned procedure with a uniform time step of 0.001 s. This time step is selected so as to compare the present results of slug displacement with that of Ma *et al.* (2006). The results obtained for the time interval of 0-0.005 s in steps of 0.001 s is shown in Table II, so that the readers can implement the steps easily.

4. Results and discussion

In the present work, the effect of additional working fluids, namely, ethanol and methanol on the displacement is studied. The variation of different forces acting on the fluid is analysed. The slug velocity is determined from the governing differential equation. Parametric studies showing the effect of parameters such as fill ratio, diameter of tube and operating temperature on the slug velocity has also been carried out. An attempt is made to characterize the PHP through non-dimensional numbers such as Poiseuille number, capillary number and Eckert number.

4.1 Comparison of displacement of the slug with the literature

Figures 2(a)-(d) show the comparison of slug displacements with respect to time obtained by the present study and the study of Ma *et al.* (2006) for various parameters. It is seen that the displacements of the slugs obtained in the present study matches exactly with the Ma *et al.*'s work.

4.2 Study of displacement of additional working fluids

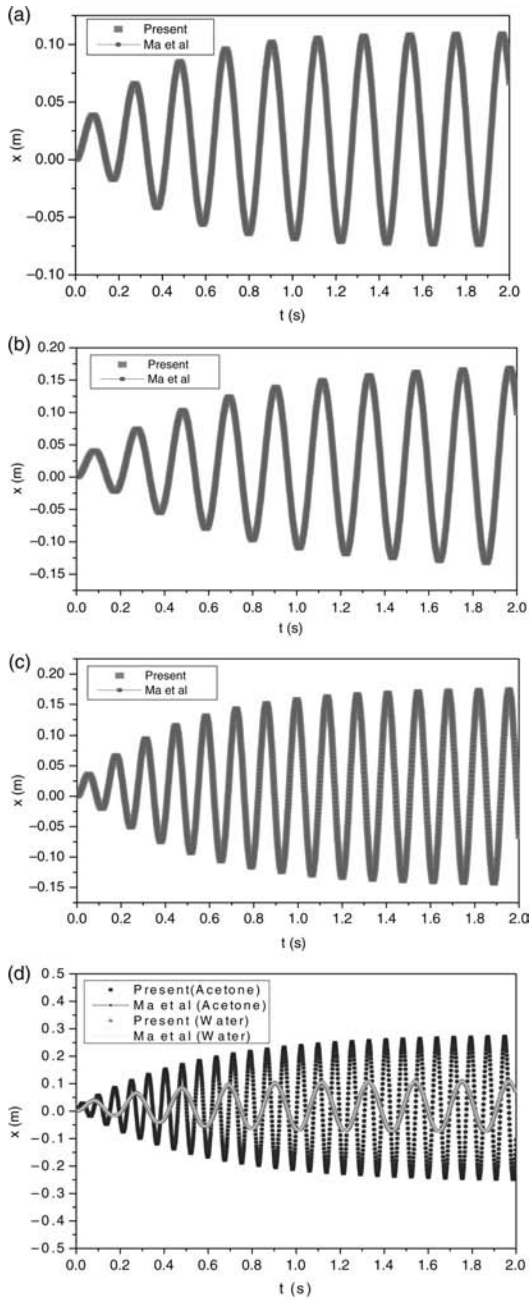
Ma *et al.* (2006) have studied the behaviour of two working fluids, namely, water and Acetone in a PHP. The present study is further extended to study the behaviour of other working fluids, namely, methanol and ethanol. The displacement plots of methanol and ethanol are shown in Figure 3. It is clear from Figure 3 that the amplitude of displacement is higher for ethanol compared to methanol whereas the frequency of oscillation is more in methanol. However, considering Figures 2(d) and 3, it can be concluded that Acetone has the maximum amplitude of displacement and frequency compared to water, ethanol and methanol.

4.3 Variation of driving and opposing forces in a PHP

The variation of different forces such as inertia, viscous, vapour pressure and driving force with respect to time for water with fill ratio of 50 per cent, diameter of 1.65 mm and operating temperature of 60 °C is shown in Figure 4. Due to the pulsating motion of

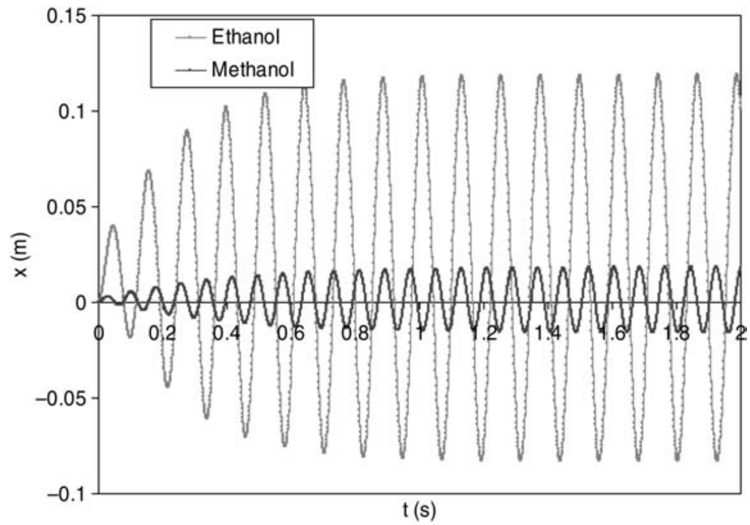
Table II. Solutions of Equations (12) and (14) from time step of 0.001-0.005 s

Time t (s)	0	0.001	0.002	0.003	0.004	0.005
Displacement x (m)	0	1.54×10^{-5}	6.13×10^{-5}	0.000138	0.000244	0.00038
Velocity dx/dt (m/s)	0	0.03067	0.061126	0.091331	0.121246	0.150833



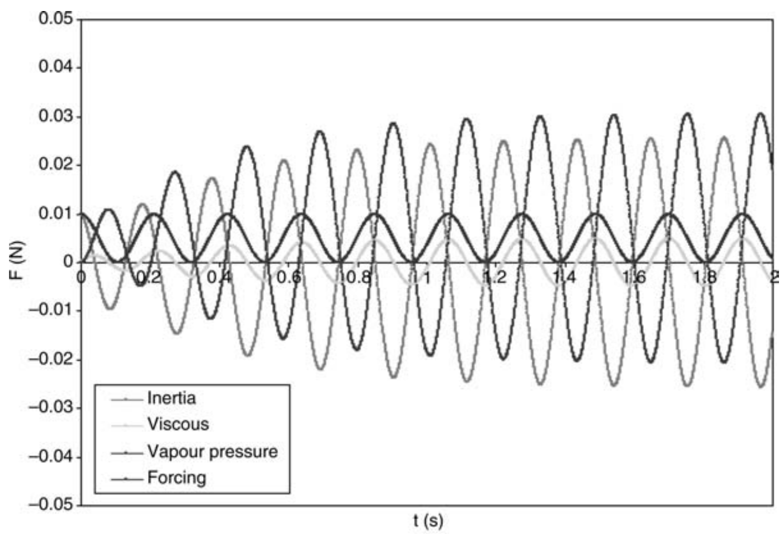
Notes: (a) Water, L = 304.8 mm, T = 60 °C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K; (b) water, L = 304.8 mm, T = 60 °C, D = 2.16 mm, Φ = 50 per cent, ΔT = 5K; (c) water, L = 304.8 mm, T = 80 °C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K; and (d) for acetone and water L = 304.8 mm, T = 60 °C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

Figure 2.
Displacement variation with time in PHP



Notes: L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

Figure 3.
Effect of ethanol and methanol on displacement



Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

Figure 4.
Variation of different forces with time

the fluid in a PHP, the variation of all the forces with respect to time is sinusoidal. The frequencies of all the forces are same but the amplitudes of forces are different. It is seen that the amplitudes of inertia and vapour pressure forces are higher than the viscous force and driving force. As there is a phase difference between the forces, the amplitude of the driving force is smaller than the amplitude of other forces. The inertia force contributes more compared to other forces.

4.4 Study of slug velocity

In the present work, the slug velocity is determined and the effect of parameters such as fill ratio, diameter of tube, operating temperature, temperature difference between evaporator and condenser and working fluid on the slug velocity is investigated.

Figure 5 shows the variation of slug velocity with respect to time for Acetone with diameter equal to 1.65 mm, fill ratio of 50 per cent and operating temperature of 60 °C. It is evident from the figure that the variation of slug velocity with respect to time in a PHP is sinusoidal. Considering that the slug velocities oscillate between mean values of zero, the RMS values of slug velocities are used for the parametric studies and for determining the non-dimensional numbers in the present work. It is shown in Figure 5 that the trends of maximum velocities (line joining the peaks) and the RMS values of velocities are similar and hence using the RMS values of velocities for parametric studies and for determining the non-dimensional numbers is reasonable.

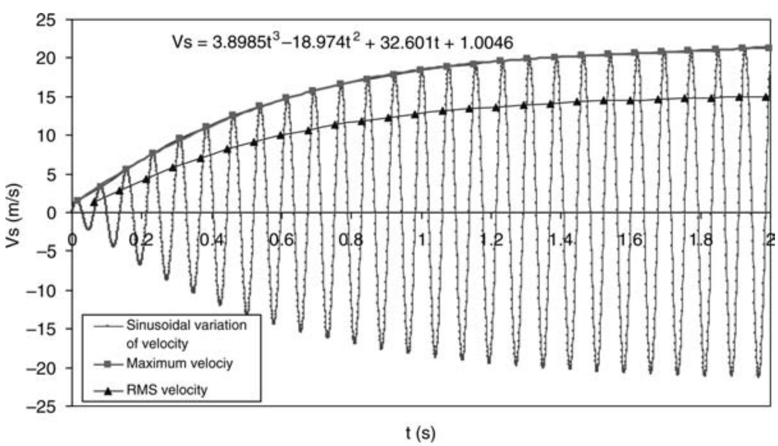
4.5 Parametric studies on slug velocity

4.5.1 Effect of fill ratio. Figure 6 shows the variation of root mean square values of velocity of the slug with respect to time for different fill ratio with water as the working fluid at diameter of 1.65 mm and operating temperature of 60 °C. Considering the pulsating nature of fluid flow in a PHP, the root mean square values of velocity are evaluated for each cycle and plotted with respect to time. The momentum of the fluid is less in the initial time steps resulting in lower slug velocity. The slug velocity increases with increase in time as the fluid gains momentum and reaches saturation at elapsed time as shown in Figure 6.

The variation of fill ratio has significant effect on the performance of PHP. Fill ratio is basically defined as:

$$\varphi = \frac{\text{Volume of Liquid}}{\text{Total Volume}} = \frac{\frac{\pi}{4} D^2 L_l}{\frac{\pi}{4} D^2 L} = \frac{L_l}{L}. \tag{15}$$

Conduction heat transfer is dominant if the fill ratio is less than 20 per cent. If the fill ratio is more than 80 per cent, the device acts as a thermosyphon. The heat pipe works



Notes: L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

Figure 5.
Comparison of maximum and RMS slug velocity in PHP for Acetone

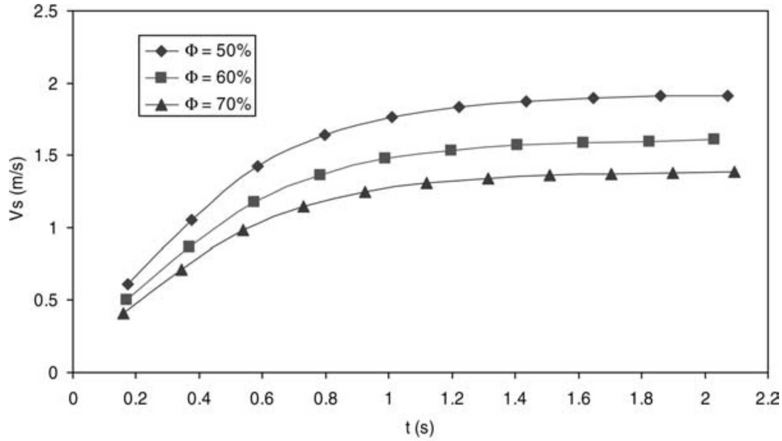


Figure 6.
Effect of fill ratio on slug velocity

Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, ΔT = 5K

as a true pulsating device in the range of 20-80 per cent fill ratio (Khandekar, 2004). It is known from literature that lower fill ratio results in the formation of more bubbles but small liquid mass for heat transport. Higher fill ratio causes fewer bubbles resulting in lesser capillary pumping action. Hence, the selection of an optimum fill ratio for better performance of PHP is critical. In the present study, the fill ratio is varied in the range of 50-70 per cent. It is observed from Figure 6 that the slug velocities are higher at lower fill ratio since the displacement of the fluid is more at lower fill ratios.

4.5.2 Effect of diameter. The variation of slug velocity with respect to time for different diameters with water as the working fluid with fill ratio of 50 per cent at operating temperature of 60 °C is shown in Figure 7. When a PHP is filled partially, the working fluid breaks into liquid slugs and vapour plugs. It is reported in the literature that the maximum diameter that can hold a vapour bubble in a PHP tube is 2.5 mm (Shafii *et al.*, 2001). In the present study, three diameters of 1.16, 1.65 and 2.16 mm are considered. Due to inertia, the velocity of the slug remains almost same at all diameters

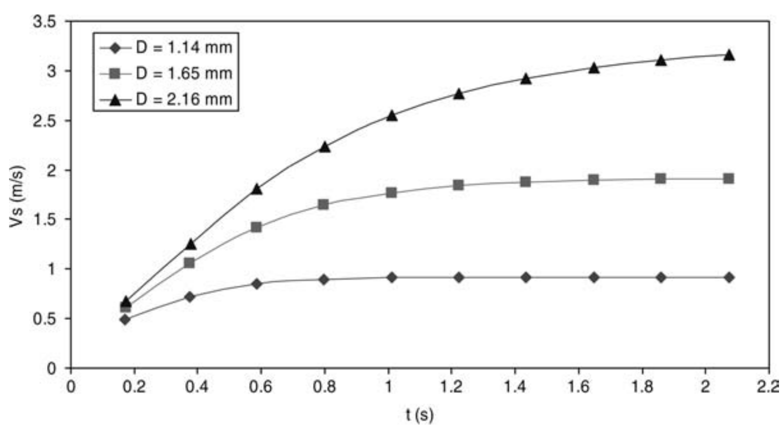


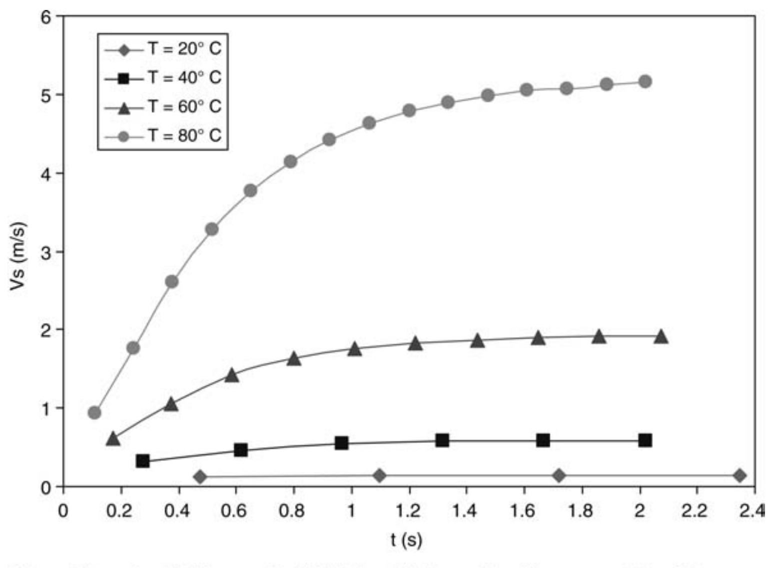
Figure 7.
Effect of diameter on slug velocity

Notes: Water, L = 304.8 mm, T = 60°C, Φ = 50 per cent, ΔT = 5K in the initial time steps. But with the increase in the momentum of the fluid, the velocity of the slug shows an increasing trend with an increase in diameter. The resistance to the fluid flow decreases with increase in diameter due to the reduction in viscous force as seen from Equation (5). Hence, higher slug velocities are observed at higher diameter. It is also seen that the system takes more time to reach steady state at higher diameter.

4.5.3 Effect of operating temperature. Figure 8 shows the variation of velocity of the slug as a function of time at different operating temperatures for water with diameter of 1.65 mm and fill ratio of 50 per cent. The thermal energy available for the momentum of the working fluid increases with increase in operating temperature. This results in higher slug velocity at higher operating temperature. It is also seen that the momentum of the fluid is very less at 20 and 40 °C as the displacement of the fluid is less due to lower thermal energy. Hence, it is desired to operate PHP at higher operating temperatures.

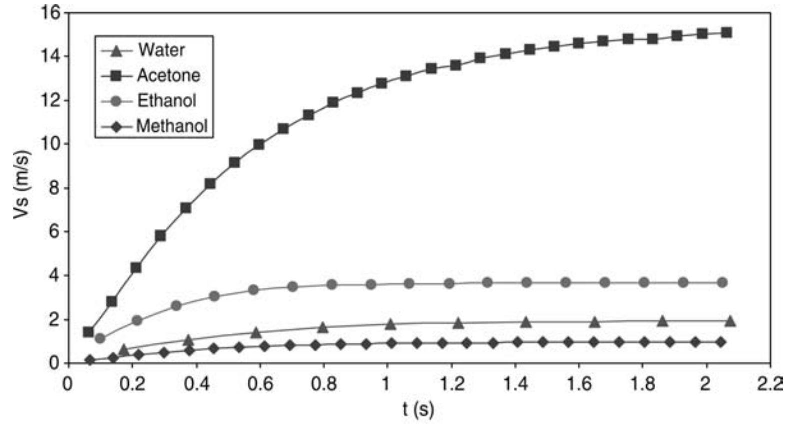
4.5.4 Effect of working fluid. The variation of RMS slug velocity with respect to time for different working fluids with diameter of 1.65 mm, fill ratio of 50 per cent and operating temperature of 60 °C is shown in Figure 9. It is observed that the magnitude of slug velocities is maximum for Acetone compared to ethanol, water and methanol. Higher values of slug velocities for Acetone can be attributed to its lower value of latent heat.

4.5.5 Effect of temperature difference between evaporator and condenser. In the present study, the temperature difference between the evaporator and condenser is taken as the driving force for the fluid motion. A variation in this value of temperature difference results in a change in the momentum transport and heat transport values. Figure 10 shows the effect of this temperature difference between evaporator and condenser on the slug velocity for water at fill ratio of 50 per cent, diameter of 1.65 mm and operating temperature of 60 °C. It is clear from Figure 10 that the slug velocities increase with increase in the temperature difference. As the energy level increases at higher temperature difference, it results in better momentum and hence better heat transport. Thus, it is



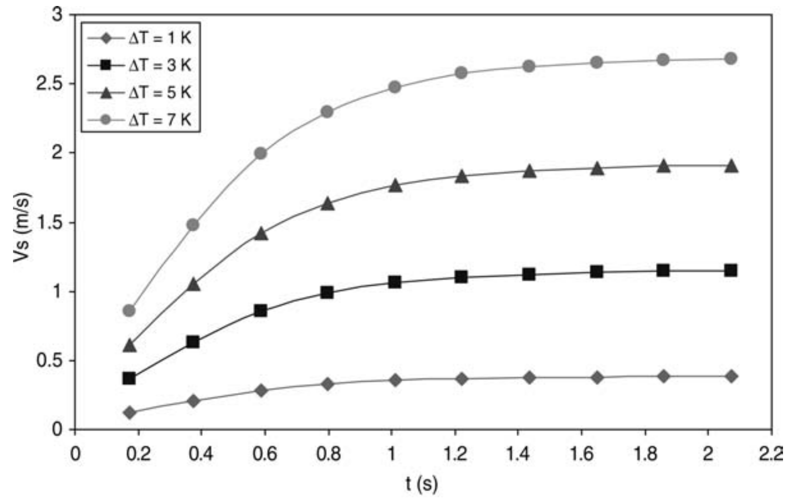
Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

Figure 8. Effect of operating temperature on slug velocity



Notes: L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

Figure 9.
Effect of working fluid on slug velocity



Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent

Figure 10.
Effect of temperature difference on slug velocity

desirable to operate the PHP with higher temperature difference between the evaporator and condenser. However, this temperature difference is limited to a smaller value as the PHP is considered to be working in the saturation region in the present study.

4.6 Significance of non-dimensional numbers in PHP

4.6.1 Poiseuille number. The Poiseuille number is basically defined as the product of Reynolds number and friction factor. Poiseuille number is one of the important non-dimensional numbers defined in micro- and mini-channel flows with low values of Reynolds number. The flow in a PHP is considered as a mini-channel flow. Hence, the analysis of oscillatory flow in a PHP with Poiseuille number is significant.

It is observed that in mini and micro channels, Poiseuille number is frequently used to study the effect of friction for single-phase flows. Higher the Poiseuille number, higher is the velocity of the slug resulting in higher momentum transfer and better heat transport. The Poiseuille number is given by (Khandekar, 2004):

$$Po = \frac{V\mu_l}{D^2 g(\rho_l - \rho_v)}. \quad (16)$$

Considering the above, the Poiseuille number is one of the important non-dimensional number in the design of PHP as it includes flow characteristics, geometry and fluid characteristics of a PHP.

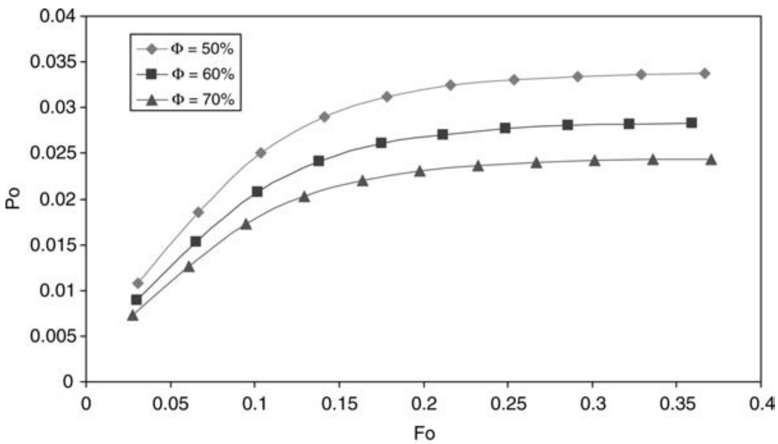
The Poiseuille number is determined from Equation (11) based on the RMS values of velocity of each cycle and plotted with respect to Fourier number which is a representation of non-dimensional time. The Fourier number for a fluid flow is given by:

$$Fo = \frac{\gamma t}{D^2}. \quad (17)$$

Figure 11 shows the variation of Poiseuille number with respect to Fourier number for different fill ratio with water as the working fluid for diameter of 1.65 mm and operating temperature of 60 °C. The Poiseuille number increases with increase in non-dimensional time due to increase in the slug velocity as indicated in Figure 6. As the fill ratio increases, the displacement and velocity of the slug becomes smaller. Hence, higher values of Poiseuille number are observed at lower fill ratio indicating better momentum transfer at lower fill ratio.

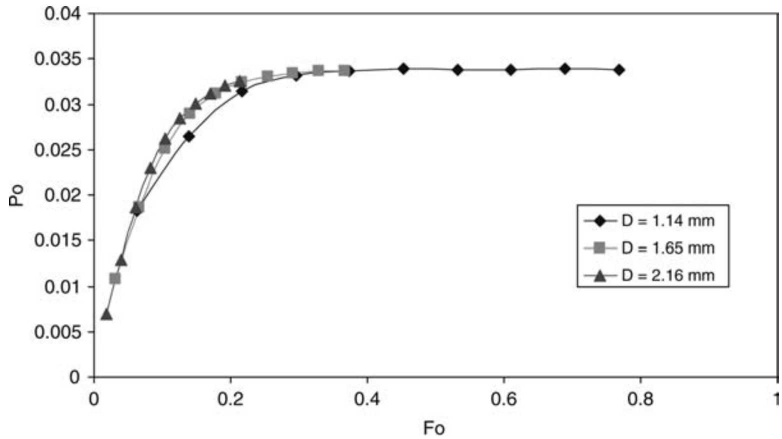
The effect of diameter on Poiseuille number with respect to Fourier number for water with fill ratio of 50 per cent and operating temperature of 60 °C is highlighted in Figure 12. The Poiseuille number increases with increase in Fourier number initially but reaches a value of 0.035 for Fourier number beyond 0.3. Hence, Poiseuille number of 0.035 can be used as the design parameter of PHP. Further, it is evident from Figure 12 that Poiseuille number is independent of diameter in steady state.

Figure 13 indicates the variation of Poiseuille number with respect to Fourier number for different operating temperatures with water as the working fluid at diameter equal to 1.65 mm and fill ratio of 50 per cent. Higher values of Poiseuille number are observed at higher operating temperatures as the available thermal energy increases the slug velocity at higher temperatures as seen in Figure 8. Higher is the



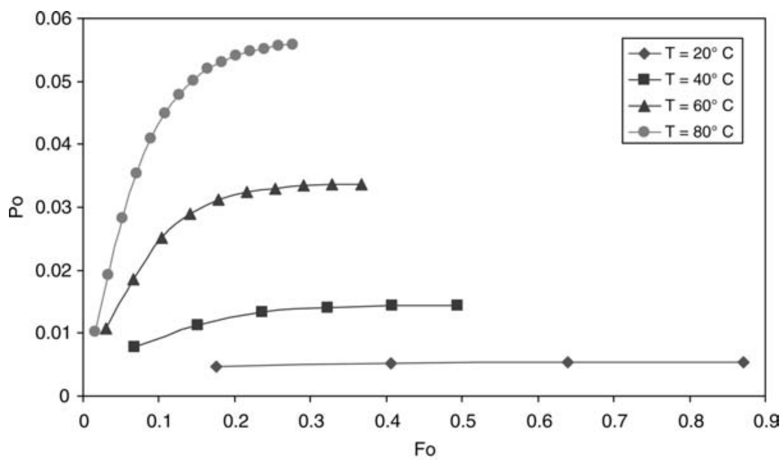
Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, ΔT = 5K

Figure 11. Effect of fill ratio on Poiseuille number



Notes: Water, L = 304.8 mm, T = 60°C, Φ = 50 per cent, ΔT = 5K

Figure 12.
Effect of diameter on Poiseuille number

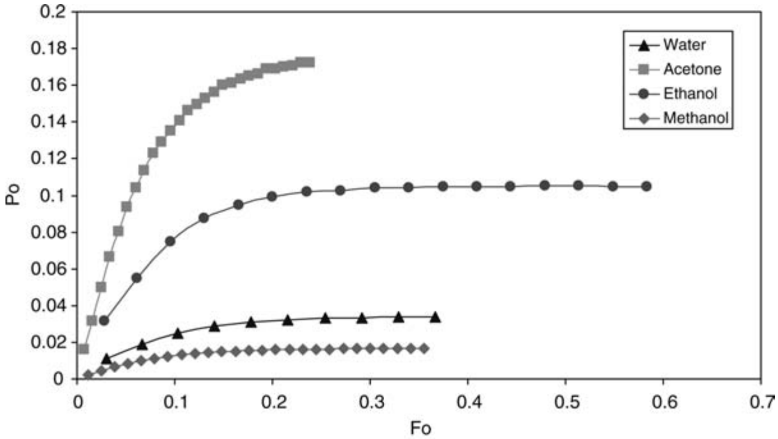


Notes: Water, L = 304.8 mm, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

Figure 13.
Effect of operating temperature on Poiseuille number

operating temperature, better will be the performance of PHP and hence higher operating temperatures are recommended. However, it is limited by the material of PHP in terms of withstanding of thermal stresses.

The variation of Poiseuille number with Fourier number for different working fluids at diameter of 1.65 mm, fill ratio of 50 per cent and operating temperature of 60 °C is shown in Figure 14. When the working fluid is changed from water to Acetone, both the displacement and velocity of the slug increase rapidly as shown in Figure 9. Due to increase in the velocity of the slug, higher values of Poiseuille number are observed in Acetone compared to other working fluids. As Acetone indicates higher Poiseuille number compared to other working fluids, it can be concluded that Acetone can exhibit better heat transport and fluid flow characteristics of PHP and hence can be considered as more suitable working fluid for PHP.

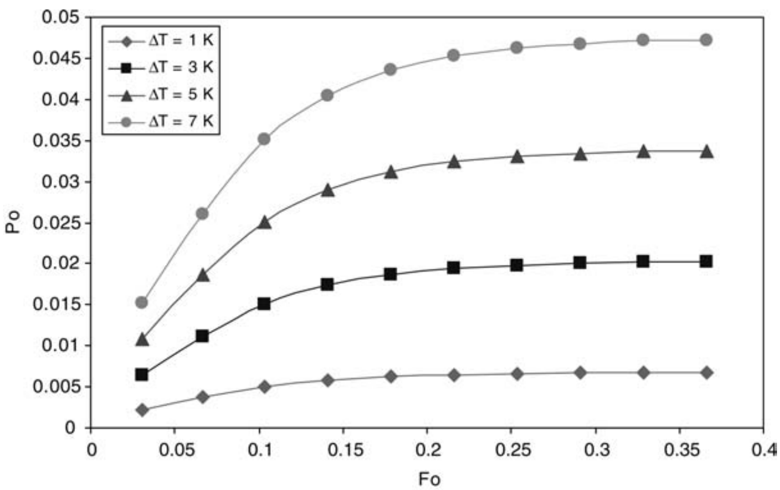


Notes: L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

Figure 14.
Effect of working fluid on Poiseuille number

Figure 15 shows the effect of temperature difference between evaporator and condenser on Poiseuille number for water at fill ratio of 50 per cent, diameter of 1.65 mm and operating temperature of 60 °C. Higher values of Poiseuille number can be observed at higher temperature difference as the slug velocities are higher at higher values of temperature difference. The temperature difference of 5-7 K shows higher values of Poiseuille number compared to 1 and 3 K indicating better momentum transfer capabilities at 5 and 7 K. Hence, it is desired to work with PHP at temperature difference of 5-7 K.

4.6.2 Capillary number. Considering that small diameter tubes are used in PHP, the effect of capillary forces assumes greater importance and hence the study of a non-dimensional number such as capillary number in PHP is recommended. Capillary number represents the ratio of relative effect of viscous forces to surface tension acting



Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent

Figure 15.
Effect of temperature difference on Poiseuille number

across an interface between a liquid and a gas or between two immiscible liquids. It is defined as:

$$Ca = \frac{\mu_l V}{\sigma}. \qquad (18)$$

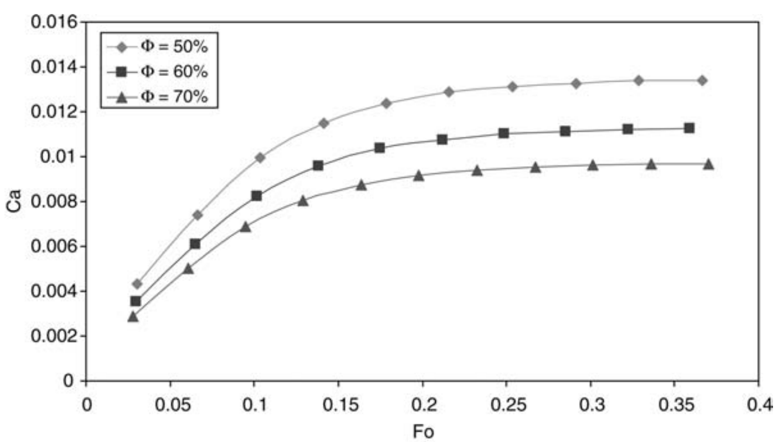
It is reported in the literatures that lower the capillary number, higher are the capillary forces.

To study the effect of fill ratio on capillary number, the variation of capillary number with Fourier number for water at different fill ratio with diameter of 1.65 mm and operating temperature of 60 °C is shown in Figure 16. The figure indicates that the capillary number increases with the reduction in fill ratio in the range of fill ratios studied. This higher value of capillary number at lower fill ratio can be attributed to higher slug velocities at lower fill ratio. This shows that the surface tension effects are little higher at higher fill ratios.

Figure 17 shows the variation of capillary number with Fourier number for different diameters of PHP with water as the fluid at fill ratio of 50 per cent and operating temperature of 60 °C. Lower values of capillary number are observed at lower diameter as the capillary forces become strong with the decrease in diameter. Since the velocity of the slug is low initially, the capillary number is almost same at all diameters in the initial time steps. As the system reaches the steady state, the velocity of the slug increases resulting in the higher value of capillary number.

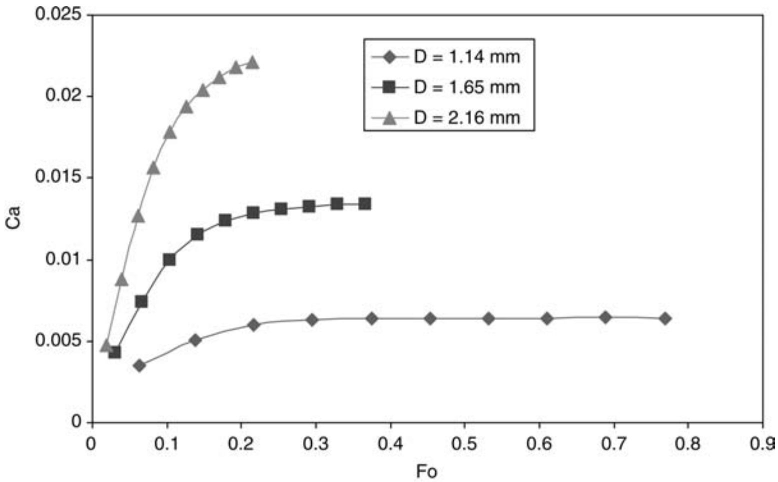
Figure 18 shows the effect of operating temperature on the capillary number with water as the fluid at diameter of 1.65 mm and fill ratio of 50 per cent. Increase in the thermal energy with increase in operating temperature results in higher slug velocity and higher capillary number. Hence, it can be concluded that the effect of capillary forces on the oscillatory motion of PHP reduces with increase in the operating temperature.

Figure 19 shows the variation of capillary number for different working fluids at fill ratio of 50 per cent, diameter of 1.65 mm and operating temperature of 60 °C. Very low values of capillary number are observed in case of water and methanol. This is due to



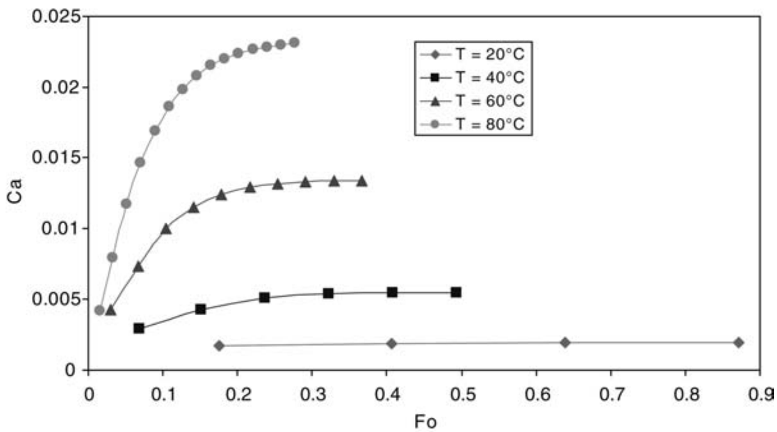
Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, ΔT = 5K

Figure 16.
Effect of fill ratio on capillary number



Notes: Water, L = 304.8 mm, T = 60°C, ΔT = 5K

Figure 17. Effect of diameter on capillary number



Notes: Water, L = 304.8 mm, T = 60°C, ΔT = 5K

Figure 18. Effect of operating temperature on capillary number

lower slug velocity and lower viscosity of these fluids. This indicates that the capillary forces are higher in water compared to Acetone.

The effect of temperature difference between evaporator and condenser on capillary number for water at fill ratio of 50 per cent, diameter of 1.65 mm and operating temperature of 60 °C is shown in Figure 20. The higher energy levels at higher temperature difference leading to higher values of slug velocities result in higher capillary number. This indicates that the effects of capillary forces are higher at lower temperature difference.

4.6.3 Eckert number. As there is a continuous evaporation and condensation of the working fluid in a PHP, a non-dimensional number which characterizes the velocity of the slug and the temperature difference between the evaporator and the condenser becomes significant. Such a dimensionless number is Eckert number. It gives the relationship between the kinetic energy of the fluid and enthalpy and is used to

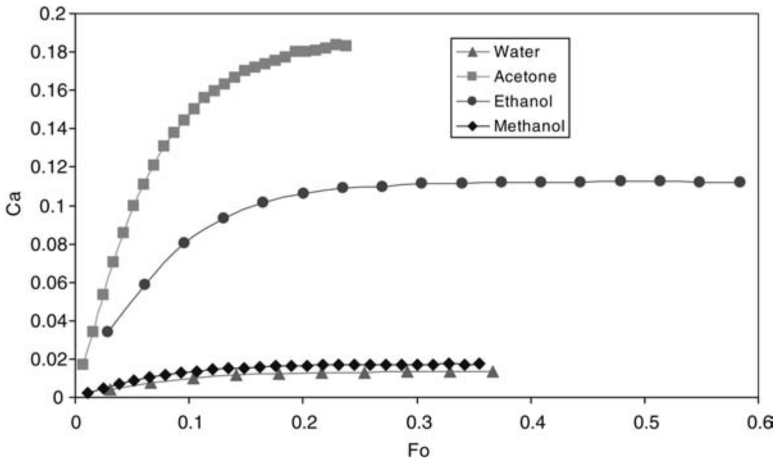


Figure 19.
Effect of working fluid on capillary number

Notes: Water, L = 304.8 mm, T = 60°C, ΔT = 5K

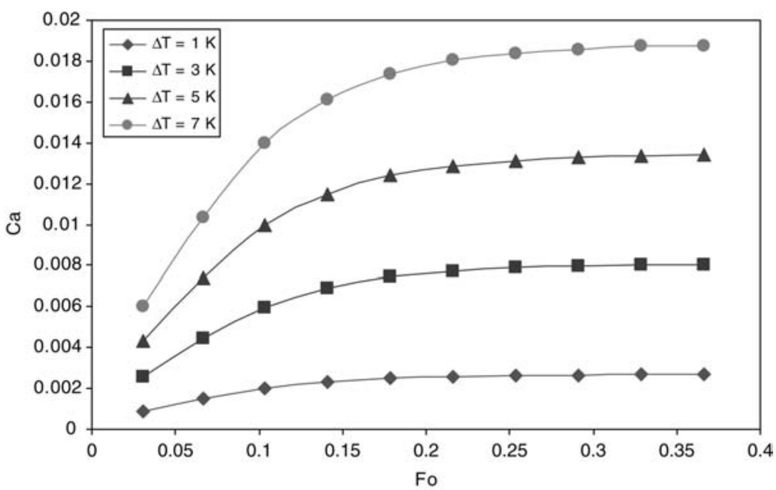


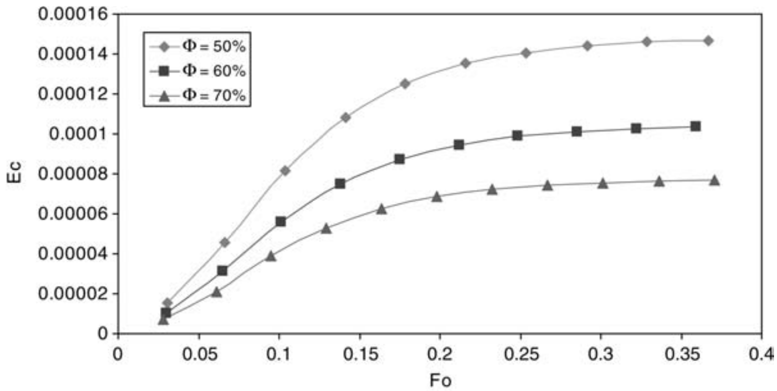
Figure 20.
Effect of temperature difference on capillary number

Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent

characterize dissipation. It is defined as:

$$Ec = \frac{V^2}{(C_{pl} + C_{pv})\Delta T}. \tag{19}$$

Figure 21 shows the variation of Eckert number with Fourier number for water at different fill ratio and diameter of 1.65 mm and operating temperature of 60 °C. There is an increase in the value of Eckert number with increase in Fourier number. It is observed that there is an increase in the values of Eckert number with decrease in fill ratio. As there is a higher displacement and slug velocity at lower fill ratio, it results in higher value of Eckert number at lower fill ratio.



Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, ΔT = 5K

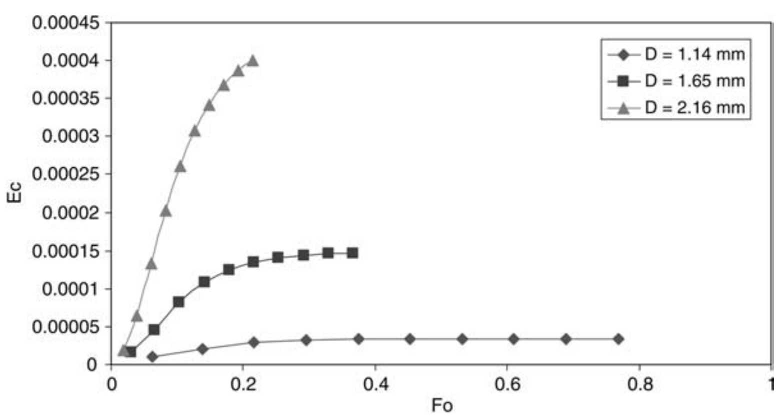
Figure 21.
Effect of fill ratio on Eckert number

The variation of Eckert number with Fourier number for different diameters with water as the fluid at fill ratio of 50 per cent and operating temperature of 60 °C is shown in Figure 22. Higher Eckert number is observed at higher diameter during the oscillatory motion in a PHP in Figure 22. As the velocity of the slug is higher at higher diameter, it results in the higher values of Eckert number.

Figure 23 shows the effect of operating temperature on the Eckert number with water as the fluid at diameter equal to 1.65 mm and fill ratio of 50 per cent. Increase in the slug velocity with increase in the operating temperature results in higher values of Eckert number at higher operating temperature.

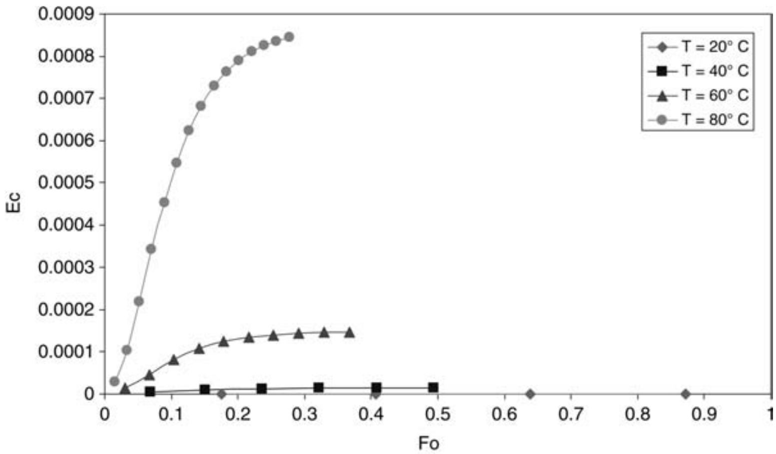
Figure 24 give the comparison in the values of Eckert number for different working fluids with fill ratio of 50 per cent, diameter of 1.65 mm and operating temperature of 60 °C. Very low values of Eckert number are observed in case of water and methanol as the slug velocity is less and the specific heat is more in water and methanol compared to Acetone.

The effect of temperature difference between evaporator and condenser on Eckert number for water at fill ratio of 50 per cent, diameter of 1.65 mm and operating temperature of 60 °C is shown in Figure 25. It is clear from the figure that higher



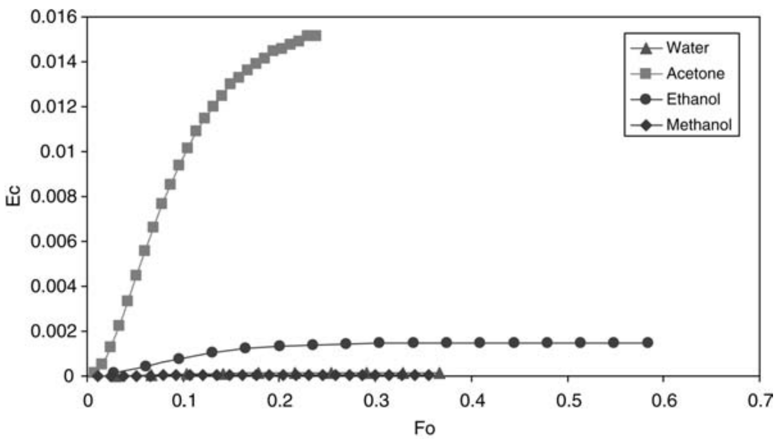
Notes: Water, L = 304.8 mm, T = 60°C, Φ = 50 per cent, ΔT = 5K

Figure 22.
Effect of diameter on Eckert number



Notes: Water, L = 304.8 mm, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

Figure 23.
Effect of operating temperature on Eckert number



Notes: L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent, ΔT = 5K

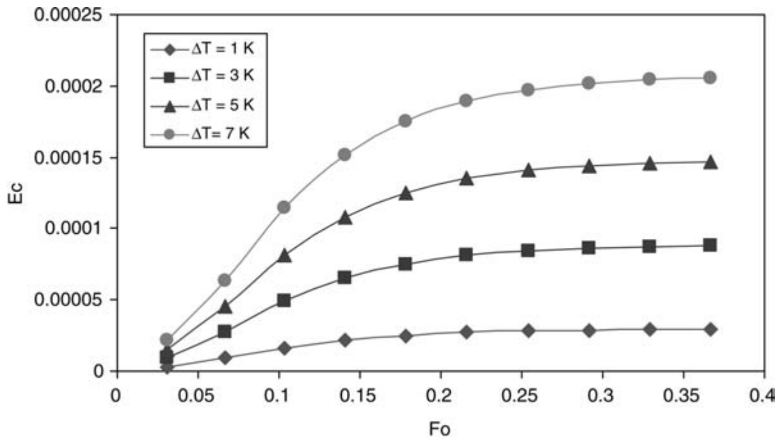
Figure 24.
Effect of working fluid on Eckert number

temperature difference between evaporator and condenser results in higher values of Eckert number.

5. Conclusions

In the present numerical investigations, the oscillating motion in a PHP is studied considering the thermal energy due to temperature difference between the evaporator and the condenser as the driving force. The governing equation is solved for the displacement and the velocity of the slug. The results are non-dimensionalized based on the RMS values of velocity of each cycle and the effect of fill ratio, diameter, operating temperature and working fluid on the dimensionless numbers is investigated. The following conclusions can be drawn from the accomplished work:

- The driving and opposing forces considered in the study have same frequency but different amplitude. The amplitude of inertia and vapour pressure force is



Notes: Water, L = 304.8 mm, T = 60°C, D = 1.65 mm, Φ = 50 per cent

Figure 25.
Effect of temperature difference on Eckert number

higher than the viscous and driving force. The inertia force is more significant compared to other forces.

- Higher slug velocities are associated with lower fill ratio, higher diameter, higher operating temperature and higher temperature difference between evaporator and condenser.
- Higher values of Poiseuille number are observed at lower fill ratio and higher operating temperature indicating better fluid flow characteristics of PHP.
- In the present study, the effect of capillary forces are found to be significant at higher fill ratio, lower diameter, lower operating temperature and lower temperature difference between evaporator and condenser.
- Very low values of Eckert number are observed in case of water and methanol compared to Acetone.
- Higher values of slug velocities are observed in case of Acetone. This result in higher values of non-dimensional numbers for Acetone compared to other fluids. This shows that Acetone exhibits better fluid flow characteristics of PHP compared to other fluids.
- Among the three non-dimensional numbers, Poiseuille number assumes more significance since it includes flow characteristics, geometry and fluid properties of a PHP.

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